A historical perspective on the development of the PWM switch

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Abstract

This review begins with the earliest appearance, in the published literature, of a sub-circuit of a dc-to-dc converter which is common to most converters and concludes with later works in which this sub-circuit is treated analytically and ultimately reduced to its final form known today as the PWM switch. Since this is my historical perspective, it is limited to my knowledge of the published literature and as such its scope may neither be sufficiently extensive nor inclusive. I will review the works of Cuk, Landsman, Tymerski, Meares up to the emergence of the PWM switch as the smallest sub-circuit of a converter with invariant structural and electrical terminal characteristics which bestow it with an invariant equivalent circuit model. Historically, the model of the PWM switch was promoted as a pedagogical tool by pointing out its similarity to the model of the transistor and its use in amplifier circuit analysis in order to render the analysis and simulation of PWM converters easily accessible to students of power electronics. It appears that this analogy to the model of the transistor has caught on well and many well known textbooks of power electronics have already adopted it.
Overview

- Cuk’s topological analysis.
- Landsman’s canonical cell.
- Tymerski’s extension of Landsman’s cell.
- Meares’ identification of the PWM cell and its transformer model.
- Vorpérian’s identification of the PWM switch, its model in continuous and discontinuous conduction modes and refinement of its model to account for parasitic elements.
- Tymerski’s extension of the PWM switch model to harmonic analysis of switching power amplifiers using Taylor series vs. Volterra functional series approach.
- Ridley’s application to current mode control.
- Does the model of the PWM switch work for all topologies?
Cuk's topological analysis

- Cyclical rotation of an inductor connected in series with a single-pole-double-throw switch.

- Cyclical rotation of a capacitor connected in parallel with a single-pole-double-throw switch.

Reference

Cuk’s topological analysis (Cont.)

Fig. 5. Generation of the buck, boost and buck-boost converters by cyclic rotation of the series connection of inductance $L$ and switch $S$.

Fig. 16. Generation of the three converters: buck with input filter, boost with output filter, and new converter by cyclic rotation of the parallel connection of capacitance $C$ and switch $S$. 
Cuk’s topological analysis (Cont.)

➢ No mathematical treatment of either sub-circuit was given, because Cuk had already invented the systematic mathematical analysis of PWM converters using the technique of State Space Averaging.

➢ These two sub-circuits were shown to have structural invariance only.

➢ Other sub-circuits were to follow later (Tymerski, Landsman)
Landsman’s canonical cell

Reference

In order to analyze this three-terminal network in the most general way, it will be assumed that none of the terminals is grounded. Voltages will be measured with respect to an external ground reference. The switch is assumed to be in position A for time $DT$ and in position B for time $D'T$, where $T$ is one period of operation and $D + D' = 1$. Conservation of charge and power yield:

\[ I_1 + I_2 + I_3 = 0 \]  
(1)

\[ V_1I_1 + V_2I_2 + V_3I_3 = 0 \]  
(2)

Lossless components have been assumed and voltages and currents are steady-state DC values with no superimposed ripple. Averaging the currents yields:

\[ I_1 = -DI_3 \]  
(3)

\[ I_2 = -D'I_3 \]  
(4)

Substituting (3) and (4) into (2) yields:

\[ -DV_1 - D'V_2 + V_3 = 0 \]  
(5)

from which all voltage transfer functions can be derived. Substitution of (3) and (4) into (1) yields:

\[ -D - D' + 1 = 0 \]  
(6)

Solving these equations.
Landsman’s canonical cell (Cont.)

**Fig. 2 Down-Converter**

**Fig. 3 Up-Converter**

**Fig. 4 Buck-Boost Converter**
Landsman’s canonical cell (Cont.)

FIG. 10 DOWN-CONVERTER

FIG. 11 UP-CONVERTER
Tymerski’s generalization of Landsman’s canonical cell $^{3,4}$

![Diagram of a three-terminal converter cell with designated terminal voltages and currents.]

Table 2.1 The six ways of configuring a three-terminal converter cell to the input source and output sink. The table entries represent the terminal number of the converter cell.

<table>
<thead>
<tr>
<th>CONFIGURATION NUMBER</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>INPUT</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>OUTPUT</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Reference


**Tymerski’s generalization of Landsman’s canonical cell** (cont.)

Many new topologies were revealed.
Meares' switching cell

The smallest invariant subcircuit is identified both structurally and quantitatively.

Figure 7, Buck regulator

\[ V_2 = D \cdot V_1, \text{ Averaged by } L \]
\[ I_1 = D \cdot I_2, \text{ Averaged by } C \]

Figure 8, Small signal Buck regulator model

\[ V_2 = D \cdot V_1 + d \cdot V_1 \]
\[ I_1 = D \cdot I_2 + d \cdot I_2 \]

Reference

Meares’ PWM cell

The PWM cell is identified and modeled as an auto-transformer with an adjustable turns ratio equal to the duty cycle.

A Spice simulation model is developed which automatically linearizes the PWM cell at a dc operating point numerically to determine the dc and small signal characteristics of the basic converters.

Reference

Meares’ PWM cell

The PWM cell model is used to analyze buck and boost converters

Reference

Meares’ PWM cell

The PWM cell model is used to analyze buckboost and Cuk converters

Reference

The model of the PWM switch

- The PWM switch and its equivalent circuit model were discovered independently by Vorperian at Virginia Tech in the same year as Meares published his APEC paper in 1986.

- The model included and accounted for the subtle effect of parasitic resistances in the PWM converter.

- The model of the PWM switch in discontinuous conduction mode operation was also given.

Reference


The model of the PWM switch (Cont.)

- It has been shown that the model of the PWM switch can be identified in any two-state dc-to-dc converter.\(^8\)

- The model has been extended to charge control and peak current mode control with constant on-time, and constant off-time.

- Simulation models were developed by various software companies such as MicroSim Pspice, Analog Workbench, and many others.

Reference

The model of the PWM switch (Cont.)
The model of the PWM switch (Cont.)
The model of the PWM switch (Cont.)

Average model of the PWM switch and small-signal model of the PWM switch.
The model of the PWM switch (Cont.)

Three immediate and significant milestones followed the development of the model of the PWM switch:

1. **Simplified analysis of PWM converters**: It was shown that the dynamical analysis of DC-to-DC converters was significantly simplified once the equivalent circuit of the converter was obtained by substituting the model of the PWM switch and Middlbrook’s techniques in Design-Oriented Analysis were applied.

   The model of the PWM switch was promoted as a pedagogical tool by pointing out its similarity to the model of the transistor and its use in amplifier circuit analysis and simulation of PWM converters became easily accessible to students of power electronics using standard and familiar tools. Many textbooks have already adopted this technique.

Reference

**The model of the PWM switch (Cont.)**

**Example:** Determination of the small-signal characteristics of the emitter follower

```
\[ \frac{v_o}{v_{in}} \equiv \text{voltage gain } TF \]
```

Replace the transistor with its equivalent circuit model.
The model of the PWM switch (Cont.)

**Example:** *Determination of the DC and small-signal characteristics of the boost converter.*

Replace the PWM switch with its equivalent circuit model:

\[
\frac{v_o}{d} \equiv \text{Control-to-output TF}
\]

\[
\frac{\hat{v}_o}{\hat{v}_g} \equiv \text{Line-to-output TF}
\]
The model of the PWM switch (Cont.)

2. Non-linear dynamical analysis of PWM converters\textsuperscript{4}: Richard Tymerski, a Ph.D. candidate at that time at Virginia Tech at that same time, recognized that he could immensely simplify the non-linear analysis of PWM switching amplifiers by replacing the Volterra functional series technique, first worked out by Robert Ericsson for PWM converters, with a simple Taylor’s series expansion of the model of the PWM switch.

Reference
The model of the PWM switch (Cont.)

2. Non-linear dynamical analysis of PWM converters

\[ \text{DC analysis} \]

\[ \text{1st harmonic} \]

\[ \text{2nd harmonic} \]

\section*{Reference}

The model of the PWM switch (Cont.)

2. **Non-linear dynamical analysis of PWM converters**

![Diagram of the PWM switch]

$N^{th}$ harmonic

**Reference**

The model of the PWM switch (Cont.)

3. **Model of current mode control**: Ray Ridley, another Ph.D. candidate at that time, immediately recognized that he could derive a continuous-time model of current-mode control simply by deriving an invariant, continuous-time model of the current feedback loop only and applying it to the model of converter obtained with the PWM switch.

Reference

The model of the PWM switch (Cont.)

3. Model of current mode control

![Diagram of PWM switch models]

(a) Buck
(b) Boost
(c) Buck-Boost

Reference

The model of the PWM switch (Cont.)

3. Model of current mode control:

<table>
<thead>
<tr>
<th>Table 4.3</th>
<th>Feedforward Gains for Constant-Frequency, Trailing-Edge Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Buck</td>
</tr>
<tr>
<td>$k_f$</td>
<td>$-DT_1R_i/L\left[1 - D/2\right]$</td>
</tr>
<tr>
<td>$k_v$</td>
<td>$T_1R_i/2L$</td>
</tr>
</tbody>
</table>

![Diagram showing power stage model](image)

Reference

The model of the PWM switch (Cont.)

3. Model of current mode control\(^7\):

The control-to-output transfer function shows a resonant peak at half the switching frequency whose damping is proportional to the external ramp and duty cycle. This correctly predicts the subharmonic oscillation of current Mode control.

Reference
The model of the PWM switch (Cont.)

Does the model of the PWM switch work for all topologies?
The model of the PWM switch (Cont.)

Does the model of the PWM switch work for all topologies?

Examine the two states of the converter and identify two PWM switches
The model of the PWM switch (Cont.)

Does the model of the PWM switch work for all topologies?

Small signal model of the converter